# Graph Structure Learning for Graph Neural Networks

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IEEE AlIOT 2022 Keynote Talk

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#### Outline

GNN Introduction

- Why Graphs?
- Graph Neural Networks: Foundations and Models

GSL Foundations

- Why Graph Structure Learning?
- Unsupervised GSL
- Supervised GSL

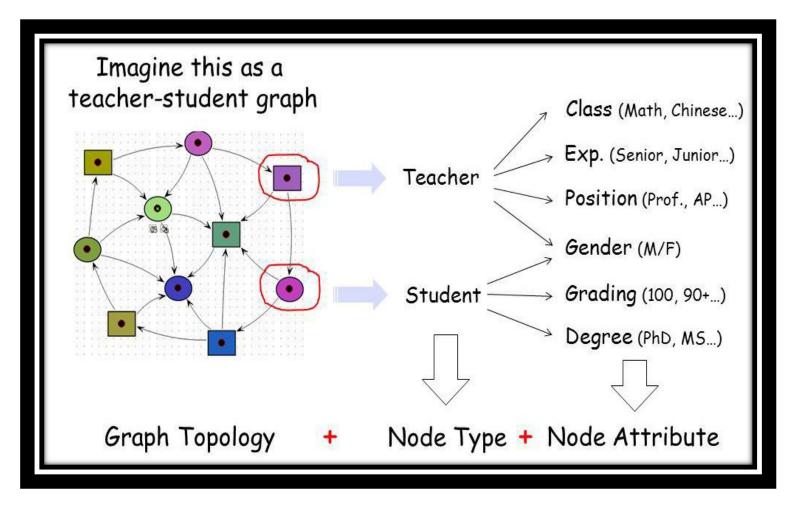
**GSL4GNN** 

- Why GSL for GNNs?
- Learning Discrete Graph Structures for GNNs
- Learning Weighted Graph Structures for GNNs
- Connections to Other Problems
- Future Directions and Conclusions

## **GNN**Introduction

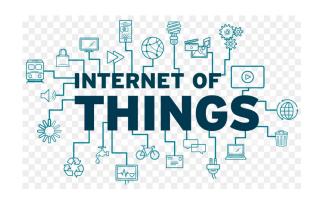
## Why Graphs?

 Graphs are a general language for describing and modeling complex systems

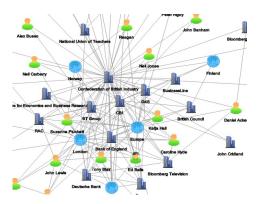


Graph = Graph Topology + Node Type + Node Attribute + Edge Type + Edge Attribute

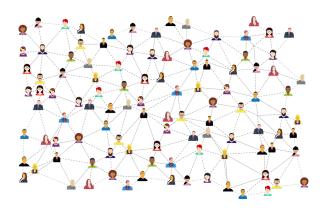
#### Graph-structured Data Are Ubiquitous



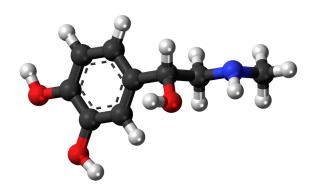
**IOT** graphs



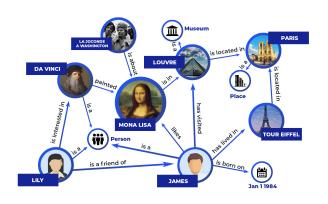
Financial transactions



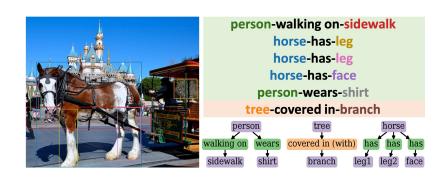
Social networks



Biomedical graphs



Knowledge graphs

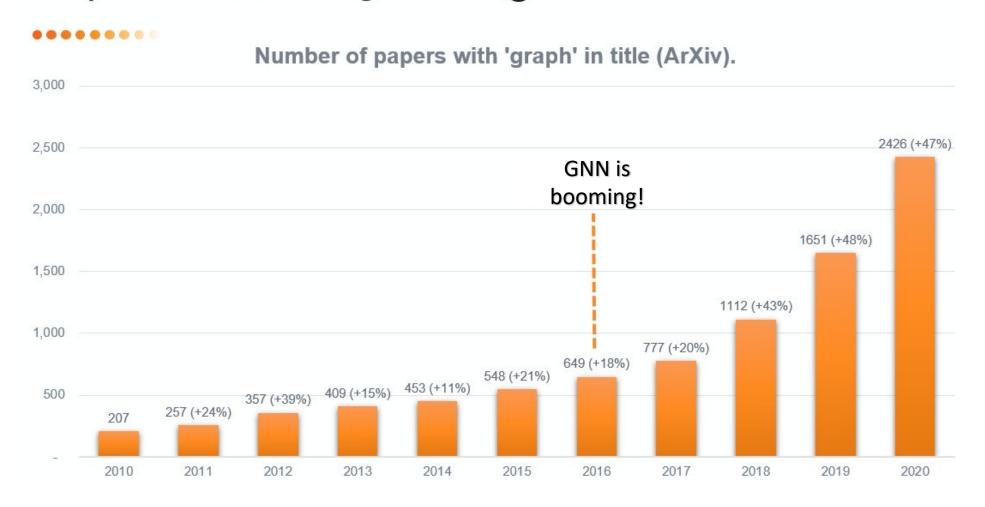


Scene graphs

Graph = Graph Topology + Node Type + Node Attribute + Edge Type + Edge Attribute

### Graph Machine Learning: Recent Trending

Graph Machine Learning is on fire 🤚



### Graph Neural Networks: A Brief History

#### Year 2016, 2017

First GRU-based Modern

**GNN** paper: **GGNN** 

First graph convolution-based paper GCN, Start a new era of **GNNs** history

#### Year 2019-2021

GNN-based applications, such as search, recommendation, Book (by Dr. L. Wu et al.): drug discovery, NLP, Transport... Many open-source libraries like DGL, Pytorch Geometric, DIG, Graph4NLP, TorchDrug...

#### **Year 2022**

Most comprehensive GNNs "Graph Neural Networks: Foundations, Frontiers, Applications" by Springer

#### **Year 2009**

First GNN Paper (Scarselli et al., 2009)

#### Year 2017-2021

A series of Graph Convolution (GCN), Message Passage (MPNN, GraphSage, GIN), Attention-based (GAT), Unsupervised GNNs (Graph-Autoencoder, graph-infomax) Many new GNNs fast developed!

#### **Year 2021**

- 3 GNN books released simultaneously:
- 1) Prof. Liu (Tsinghua) et al.:

"Introduction to Graph Neural Networks"

- 2) Prof. Tang (MSU) et al.:
  - "Deep learning on graphs"
- 3 ) Prof. Hamilton (McGill):

Graph representation learning

#### Machine Learning on Graphs Tasks

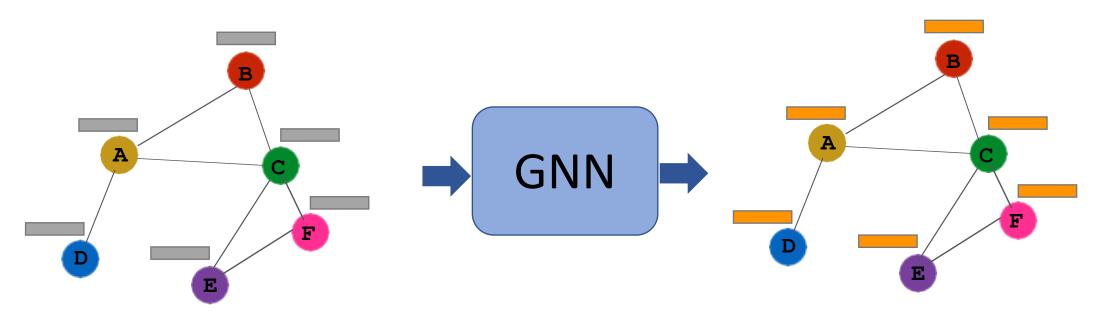
#### Classical ML tasks on graphs:

- Node classification
  - Predict a type of a given node
- Link prediction
  - Predict whether two nodes are linked
- Community detection
  - Identify densely linked clusters of nodes
- Graph similarity
  - How similar are two (sub)graphs

#### Recent ML tasks on graphs:

- Expressive power of GNNs
  - Theoretical understanding
- Scalability of GNNs
  - Sampling paradigms for scaling up
- Adversarial robustness of GNNs
  - Adversarial attacks and provable robustness
- Graph structure learning for GNNs
  - Learning optimal graph structures for GNNs

#### Modeling Graphs with Graph Neural Networks

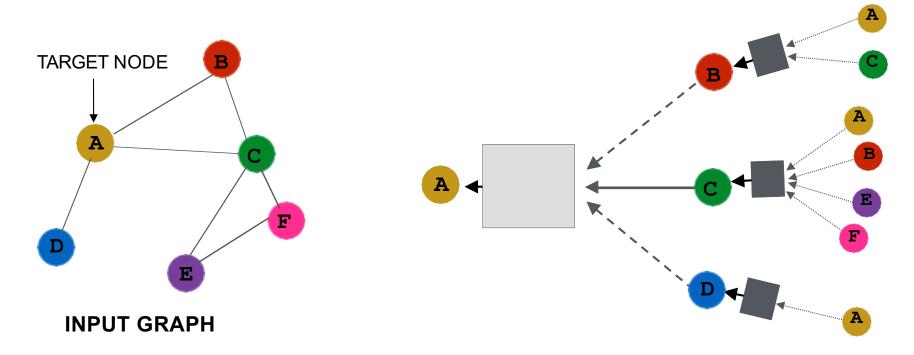


Input representations of nodes/edges

Updated representations of nodes/edges

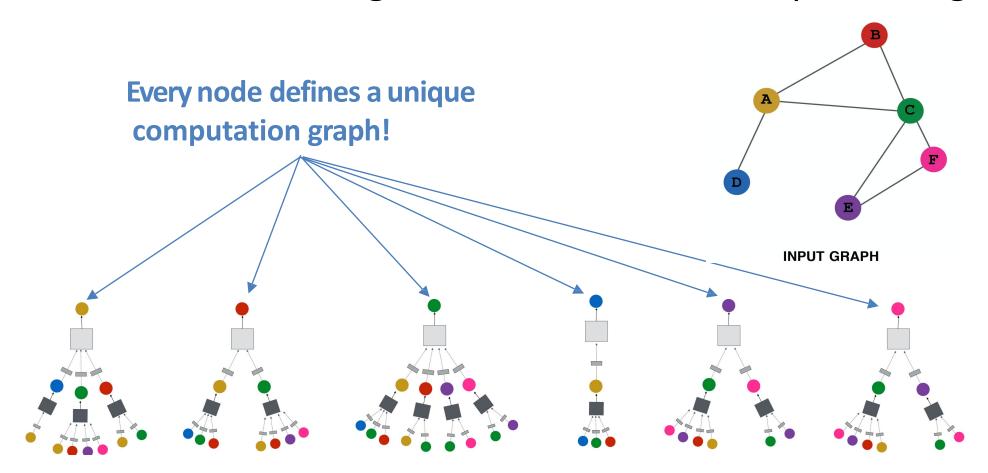
#### Graph Neural Networks: Basic Model

 Key idea: Generate node embeddings based on local neighborhoods.



## **GNN Model: Neighborhood Aggregation**

• Intuition: Network neighborhood defines a computation graph



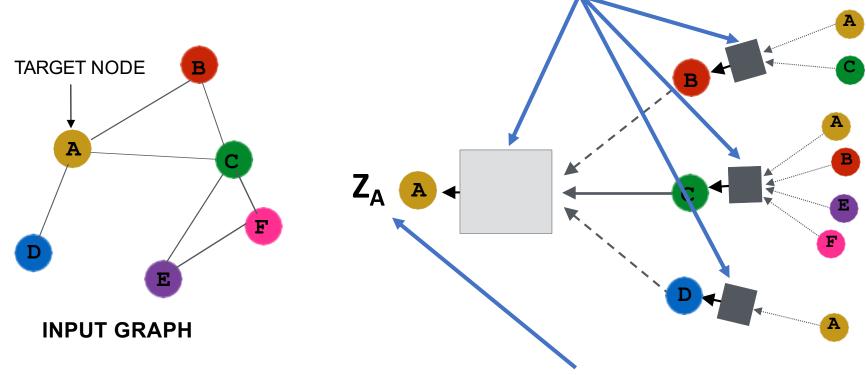
## **GNN Model: Neighborhood Aggregation**

- Nodes have embeddings at each layer.
- Model can have arbitrary depth.

"layer-0" embedding of node i is its input feature, i.e. xi. Layer-0  $X_A$ Layer-1 **TARGET NODE**  $X_{C}$ Layer-2  $X_A$  $X_{B}$ **INPUT GRAPH** 

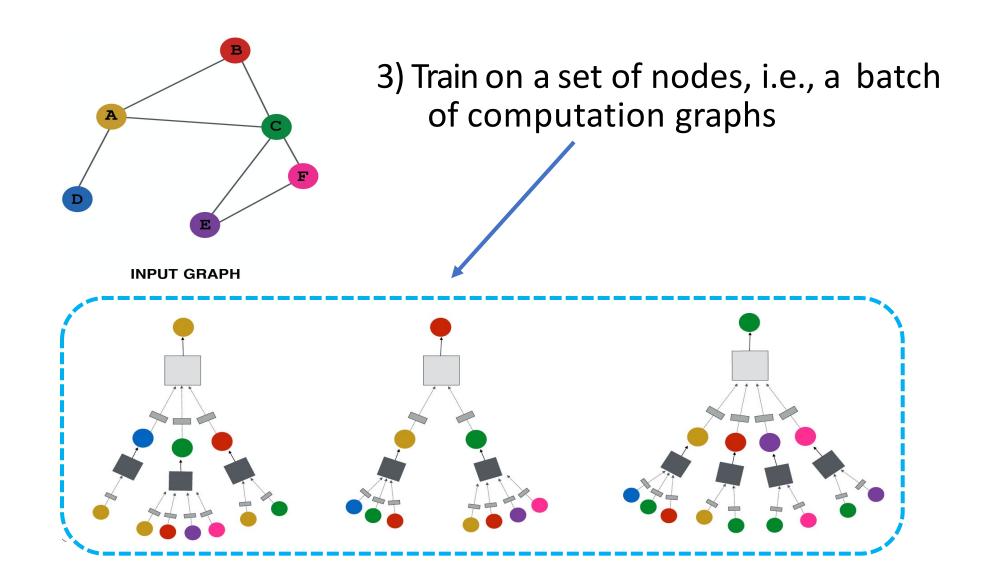
#### Overview of GNN Model

1) Define node aggregation and update functions

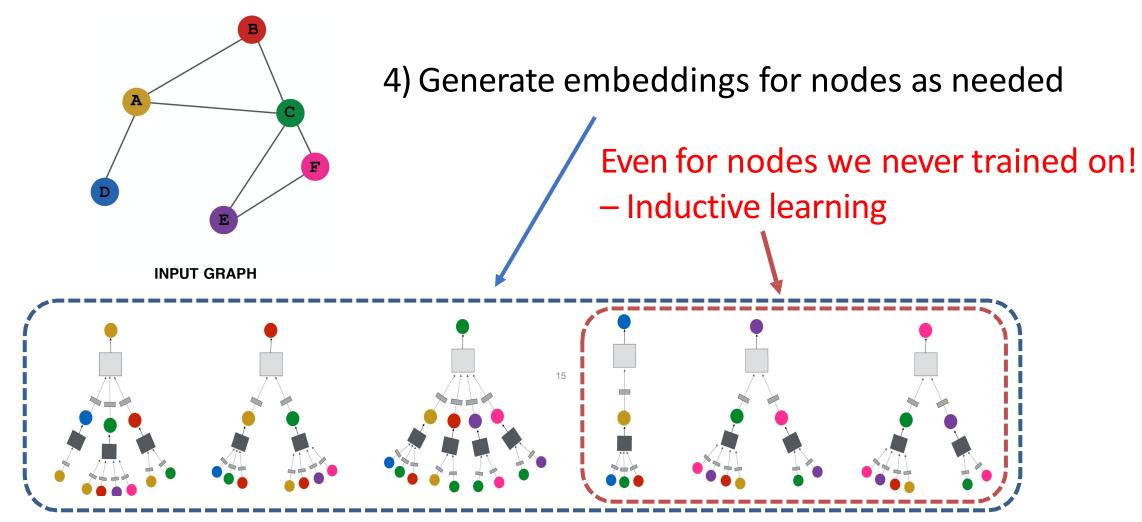


2) Define a loss function on the embeddings,  $L(z_v)$ 

#### Overview of GNN Model

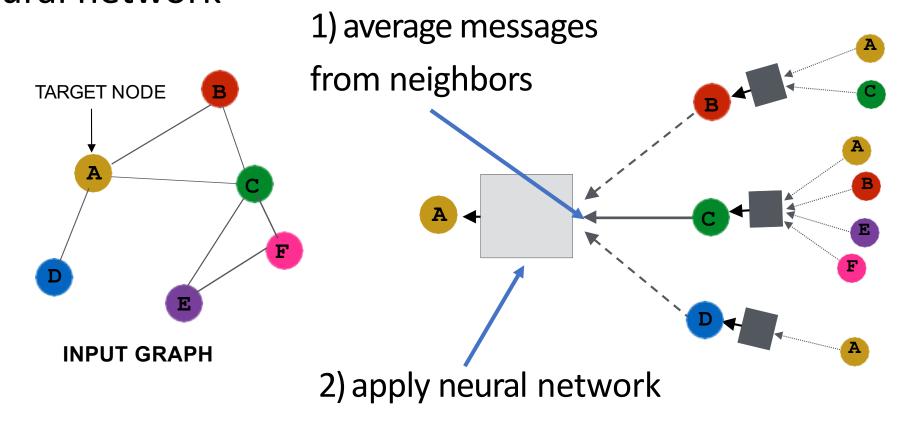


#### Overview of GNN Model



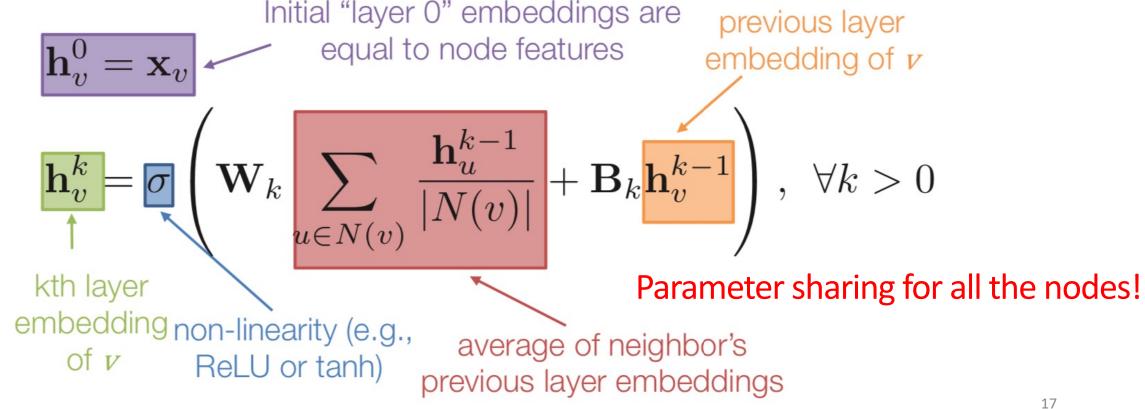
## **GNN Model: A Case Study**

 Basic approach: Average neighbor information and apply a neural network



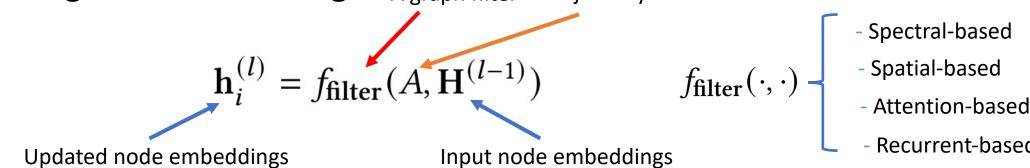
#### **GNN Model: A Case Study**

 Basic approach: Average neighbor information and apply a neural network.

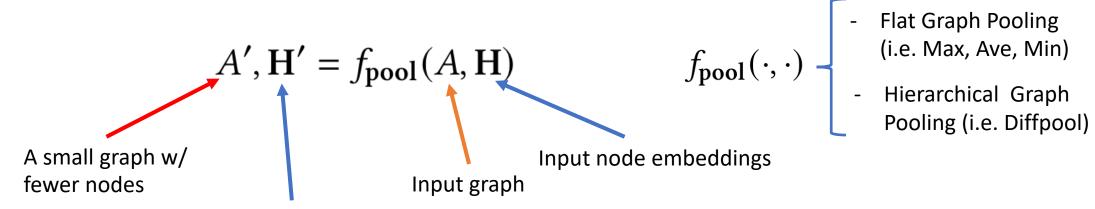


#### **Graph Neural Networks: Foundations**

• Learning node embeddings: A graph filter adjacency matrix



Learning graph-level embeddings:



New node embeddings

#### Graph Neural Networks: Popular Models

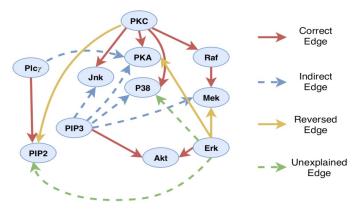
- Spectral-based Graph Filters
  - GCN (Kipf & Welling, ICLR 2017), Chebyshev-GNN (Defferrard et al. NIPS 2016)
- Spatial-based Graph Filters
  - MPNN (Gilmer et al. ICML 2017), GraphSage (Hamilton et al. NIPS 2017)
  - **GIN** (Xu et al. ICLR 2019)
- Attention-based Graph Filters
  - GAT (Velickovic et al. ICLR 2018)
- Recurrent-based Graph Filters
  - GGNN (Li et al. ICLR 2016)

#### **GNN Model: Quick Summary**

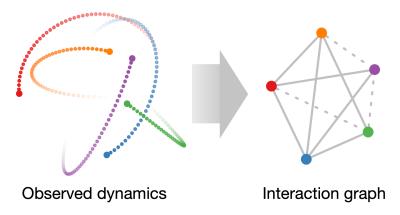
- Key idea: generate node embeddings by aggregating neighborhood information.
  - Allows for parameter sharing in the encoder
  - Allows for inductive learning

## **GSL** Foundations

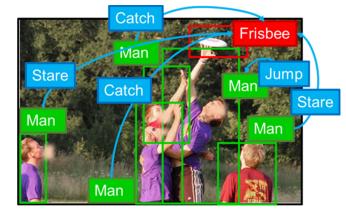
## Why Graph Structure Learning?



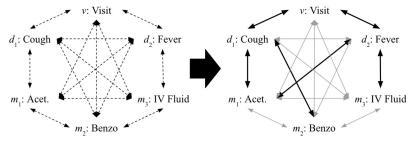
Estimating protein signaling network (Yu et al., ICML 2019)



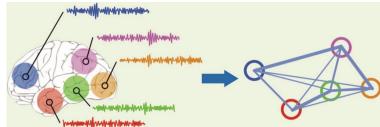
Relational inference for interacting systems (Kipf et al., ICML 2018)



Learning relationships among visual objects (Zhu et al., Multimedia Tools and Applications 2020)

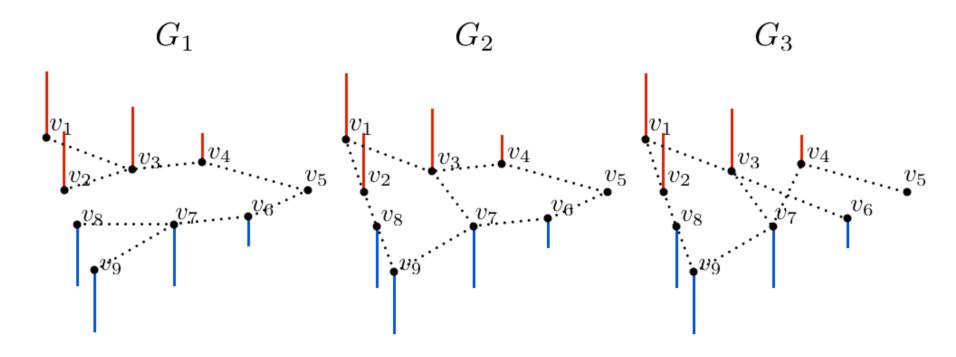


Learning the graphical structure of electronic health records (Choi et al., AAAI 2020)



Inferring functional connectivity between different brain regions (Dong et al., IEEE Signal Processing Magazine 2019)

#### Unsupervised GSL from Smooth Signals



Signals residing on graphs, graph G1 has the best smoothness property.

#### Unsupervised GSL from Smooth Signals: Fitness

OR

Node feature reconstruction using neighboring node features

$$\sum_{i} ||\mathbf{X}_{i} - \sum_{j} A_{i,j} \mathbf{X}_{j}||^{2}$$
where  $\sum_{i} A_{i,i} = 1$ ,  $A_{i,i} \ge 0$ 

where 
$$\sum_{i} A_{i,j} = 1, A_{i,j} \ge 0$$

Weighted sum of the squared distance from each node to the weighted average of its neighbors

$$\sum_{i} ||D_{i,i}\mathbf{X}_{i} - \sum_{j} A_{i,j}\mathbf{X}_{j}||^{2} = ||\mathbf{L}\mathbf{X}||_{F}^{2}$$

$$where \ D_{i,i} = \sum_{j} A_{i,j}$$

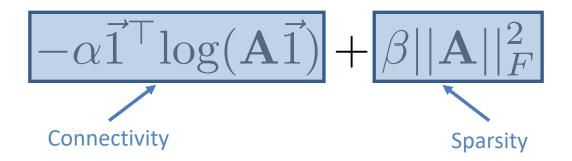
Wang et al. "Label propagation through linear neighborhoods". IEEE Transactions on Knowledge and Data Engineering 2007.

#### Unsupervised GSL from Smooth Signals: Smoothness

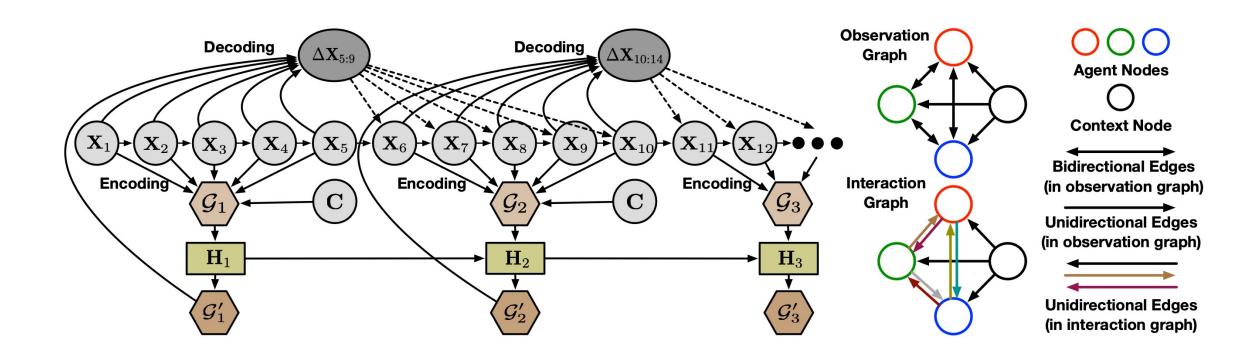
$$\Omega(\mathbf{A}, \mathbf{X}) = \frac{1}{2} \sum_{i,j} A_{i,j} ||\mathbf{X}_i - \mathbf{X}_j||^2 = \operatorname{tr}(\mathbf{X}^{\top} \mathbf{L} \mathbf{X})$$

Forcing neighboring vertices to have similar features

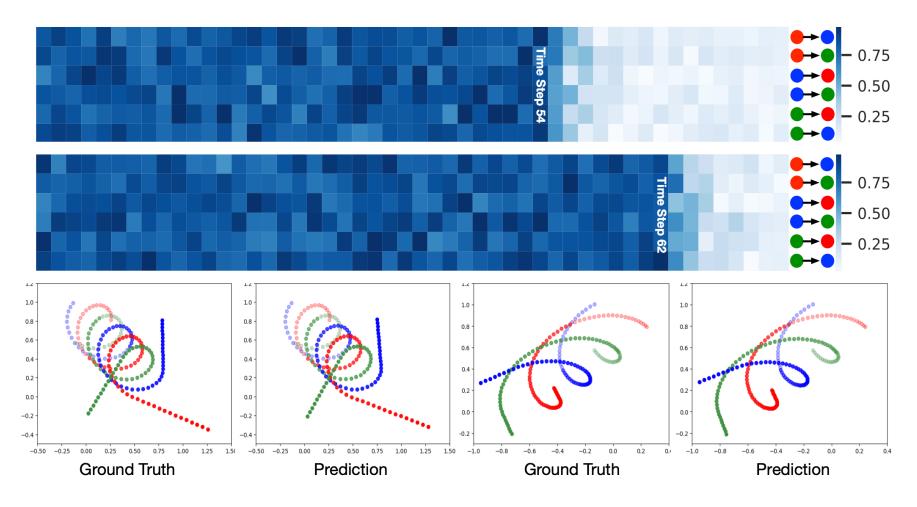
## Unsupervised GSL from Smooth Signals: Connectivity and Sparsity



#### Supervised GSL for Interacting Systems [Li et al., NeurlPS 2020]



#### Supervised GSL for Interacting Systems [Li et al., NeurIPS 2020]

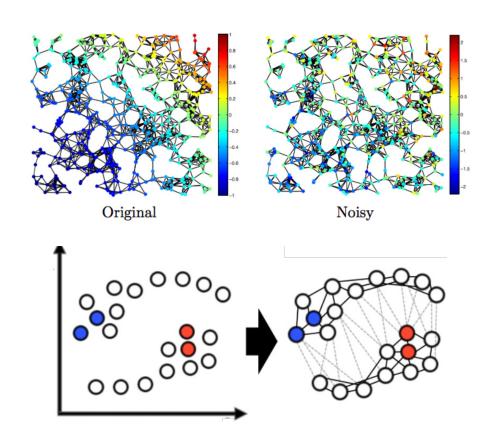


Visualization of latent interaction graph evolution and particle trajectories.

## **GSL4GNN Foundations**

#### Why GSL for GNNs?

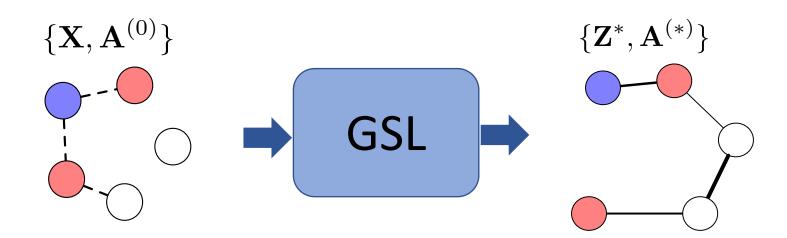
- GNNs are powerful, unfortunately, it requires graph-structured data available.
- Questionable if the given intrinsic graphstructures are optimal (i.e., noisy, incomplete, etc.) for downstream tasks.
- Many applications (e.g., NLP tasks) may only have non-graph structured data or even just the original feature matrix, requiring additional graph construction.



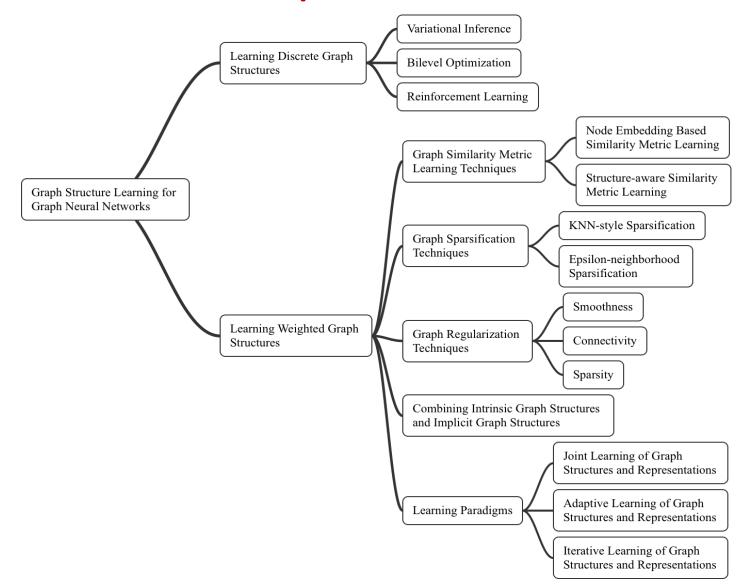
#### **GSL4GNN Formulation**

**Input:** a set of n nodes associated with a feature matrix  $\mathbf{X} \in \mathbb{R}^{d \times n}$  and an (optional and potentially noisy) initial adjacency matrix  $\mathbf{A}^{(0)} \in \mathbb{R}^{n \times n}$ .

**Output:** an optimized adjacency matrix  $\mathbf{A}^{(*)} \in \mathbb{R}^{n \times n}$  and node embedding matrix  $\mathbf{Z}^* \in \mathbb{R}^{d' \times n}$  with respect to downstream task (i.e., task-dependent loss).



### **GSL4GNN Roadmap**

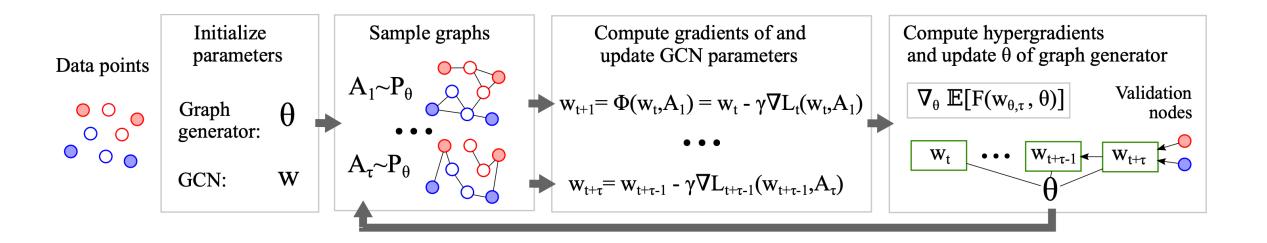


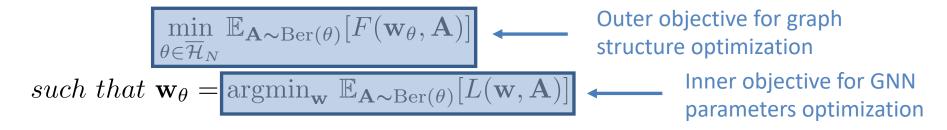
## Learning Discrete Graph Structures for GNNs

#### Learning Discrete Graph Structures for GNNs

- Sampling a discrete graph structure from learned probabilistic adjacency matrix.
- Joint graph structure and GNN parameters optimization (nondifferentiable, intractable to solve exactly) via
  - Variational inference
  - Bilevel optimization
  - Reinforcement Learning
- Non-trivial to extend to inductive learning setting.

## Bilevel Optimization for GSL [Franceschi et al., ICML 2019]

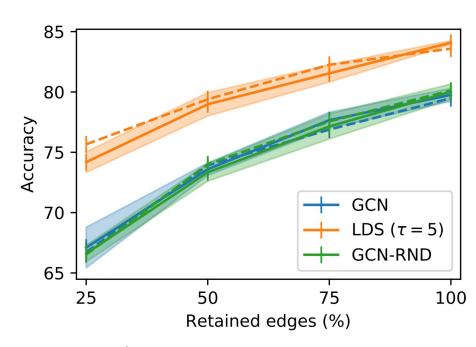




## Bilevel Optimization for GSL [Franceschi et al., ICML 2019]

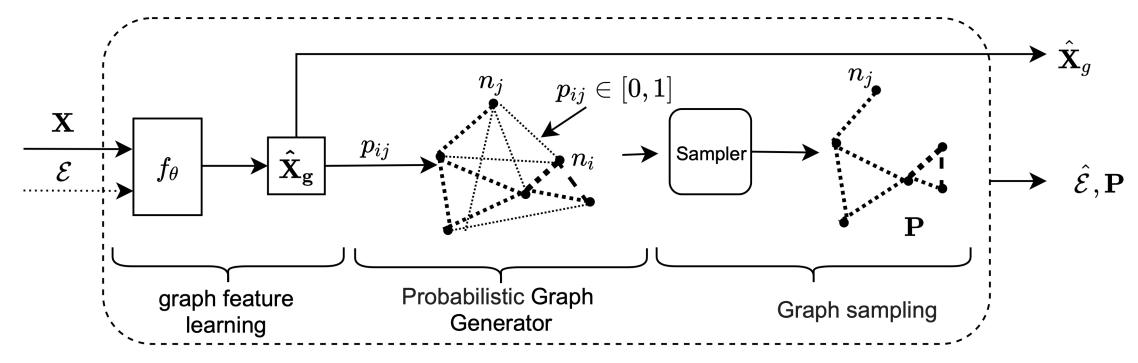
	Wine	Cancer	Digits	Citeseer	Cora	20news	FMA
LogReg	92.1 (1.3)	93.3 (0.5)	85.5 (1.5)	62.2 (0.0)	60.8 (0.0)	42.7 (1.7)	37.3 (0.7)
Linear SVM	93.9 (1.6)	90.6 (4.5)	87.1 (1.8)	58.3 (0.0)	58.9 (0.0)	40.3 (1.4)	35.7 (1.5)
RBF SVM	94.1 (2.9)	91.7 (3.1)	86.9 (3.2)	60.2 (0.0)	59.7 (0.0)	41.0 (1.1)	38.3 (1.0)
RF	93.7 (1.6)	92.1 (1.7)	83.1 (2.6)	60.7 (0.7)	58.7 (0.4)	40.0 (1.1)	37.9 (0.6)
FFNN	89.7 (1.9)	92.9 (1.2)	36.3 (10.3)	56.7 (1.7)	56.1 (1.6)	38.6 (1.4)	33.2 (1.3)
LP	89.8 (3.7)	76.6 (0.5)	91.9 (3.1)	23.2 (6.7)	37.8 (0.2)	35.3 (0.9)	14.1 (2.1)
ManiReg	90.5 (0.1)	81.8 (0.1)	83.9 (0.1)	67.7 (1.6)	62.3 (0.9)	46.6 (1.5)	34.2 (1.1)
SemiEmb	91.9 (0.1)	89.7 (0.1)	90.9 (0.1)	68.1 (0.1)	63.1 (0.1)	46.9 (0.1)	34.1 (1.9)
Sparse-GCN	63.5 (6.6)	72.5 (2.9)	13.4 (1.5)	33.1 (0.9)	30.6 (2.1)	24.7 (1.2)	23.4 (1.4)
Dense-GCN	90.6 (2.8)	90.5 (2.7)	35.6 (21.8)	58.4 (1.1)	59.1 (0.6)	40.1 (1.5)	34.5 (0.9)
RBF-GCN	90.6 (2.3)	92.6 (2.2)	70.8 (5.5)	58.1 (1.2)	57.1 (1.9)	39.3 (1.4)	33.7 (1.4)
kNN-GCN	93.2 (3.1)	93.8 (1.4)	91.3 (0.5)	68.3 (1.3)	66.5 (0.4)	41.3 (0.6)	37.8 (0.9)
kNN-LDS (dense) kNN-LDS	97.5 (1.2) 97.3 (0.4)	94.9 (0.5) 94.4 (1.9)	92.1 (0.7) 92.5 (0.7)	70.9 (1.3) 71.5 (1.1)	70.9 (1.1) 71.5 (0.8)	45.6 (2.2) 46.4 (1.6)	38.6 (0.6) 39.7 (1.4)

Test accuracy (in percentage) on various node classification datasets.



Val/test accuracy (in percent) for the edge deletion scenarios on Cora.

## Reinforcement Learning for GSL [Kazi et al., arXiv 2020]



Graph generator:

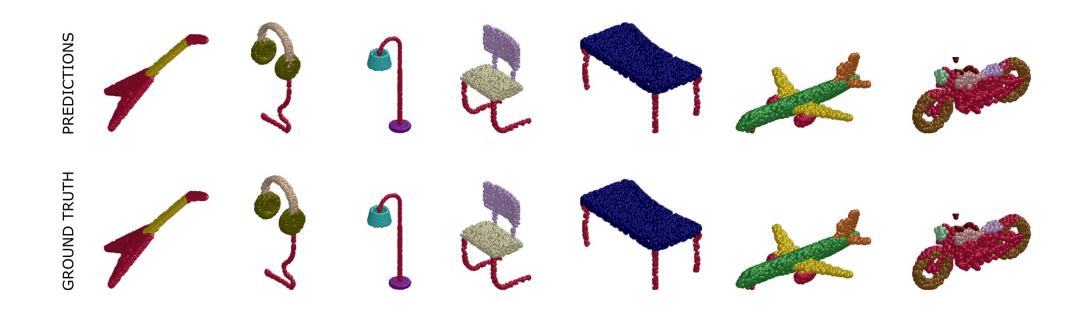
$$p_{i,j} = e^{-t||\mathbf{X}_i - \mathbf{X}_j||}$$

RL reward:

$$L_{graph} = \sum_{lpha}^{Classes} \sum_{i \in lpha} \delta_{lpha}(y_i, ilde{y}_i) \prod_{l=1}^{L} \prod_{j:(i,j) \in \hat{\mathcal{E}}^{(l)}} p_{ij}^{(l),j}$$

$$\delta_{\alpha}(y_i, \tilde{y}_i) = \begin{cases} \operatorname{acc}_{\alpha} - 1 & \text{if } y_i = \tilde{y}_i \\ \operatorname{acc}_{\alpha} & \text{otherwise} \end{cases}$$

## Reinforcement Learning for GSL [Kazi et al., arXiv 2020]



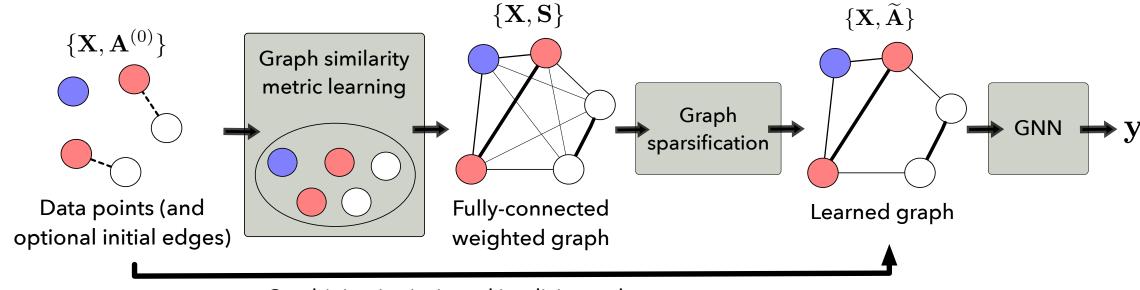
Point cloud segmentation results on ShapeNet dataset.

# Learning Weighted Graph Structures for GNNs

#### Learning Weighted Graph Structures for GNNs

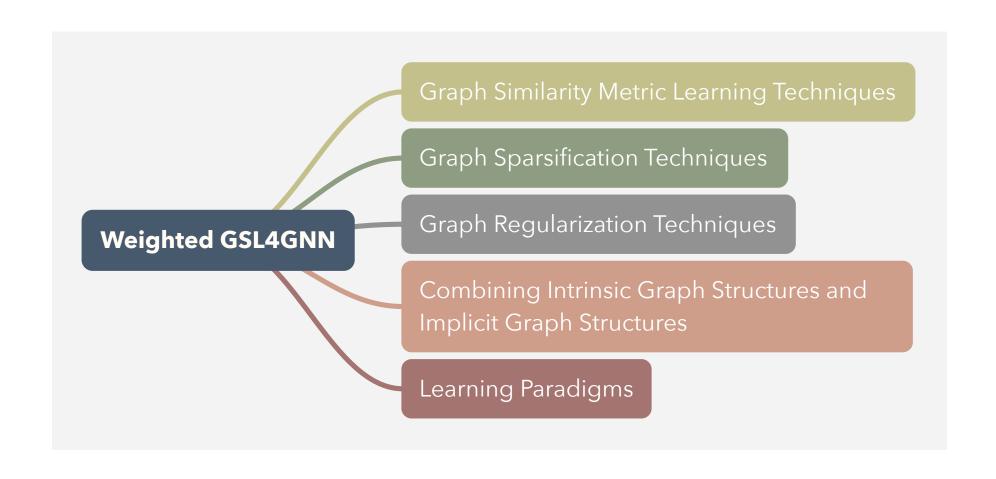
- Learning a weighted adjacency matrix to represent graph structure.
- Joint graph structure and GNN parameters optimization (differentiable, more tractable) via SGD techniques.
- Weighted adjacency matrix captures richer information.
- Handling both transductive and inductive learning settings.

## Weighted GSL4GNN Overview



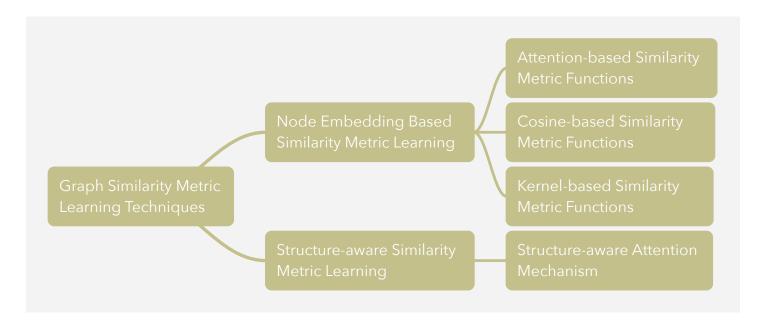
Combining intrinsic and implicit graph structures

## Weighted GSL4GNN Outline



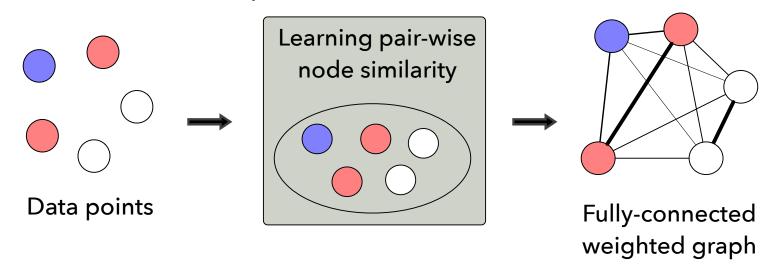
### Graph Similarity Metric Learning Techniques

- Graph structure learning as similarity metric learning (in the node embedding space)
- Enabling inductive learning
- Various metric functions



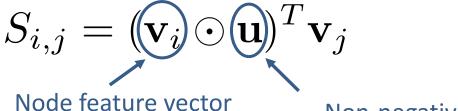
## Node Embedding Based Similarity Metric Learning

- Learning a weighted adjacency matrix by computing the pair-wise node similarity in the embedding space
- Common metrics functions
  - Attention-based similarity metric functions
  - Cosine-based similarity metric functions
  - Kernel-based similarity metric functions

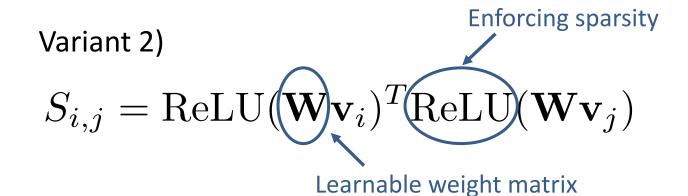


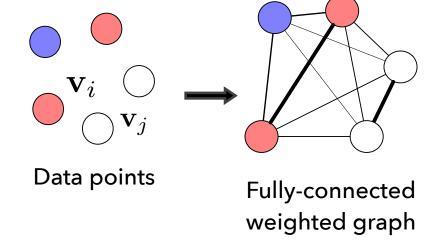
#### Attention-based Similarity Metric Functions

#### Variant 1)



Non-negative learnable weight vector

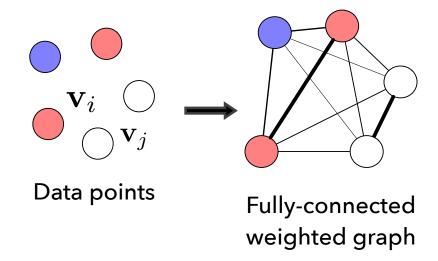




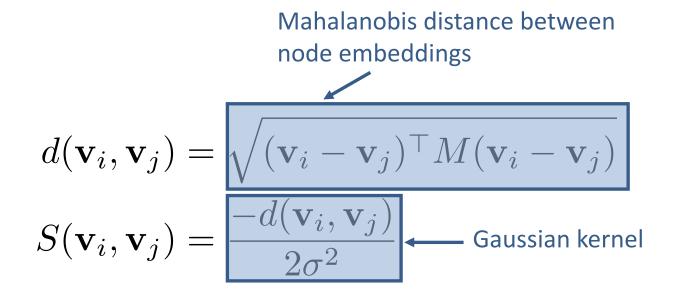
### Cosine-based Similarity Metric Functions

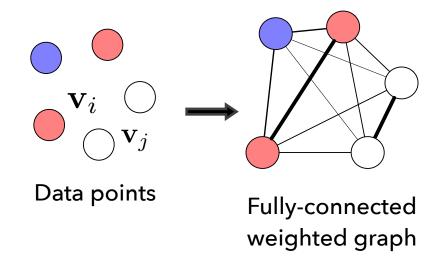
$$S_{i,j}^p = \cos(\mathbf{w}_p) \odot \mathbf{v}_i, \mathbf{w}_p \odot \mathbf{v}_j)$$
Learnable weight vector

$$S_{i,j} = oxedsymbol{rac{1}{m} \sum_{p=1}^{m} S_{ij}^p}$$
 — Multi-head similarity scores



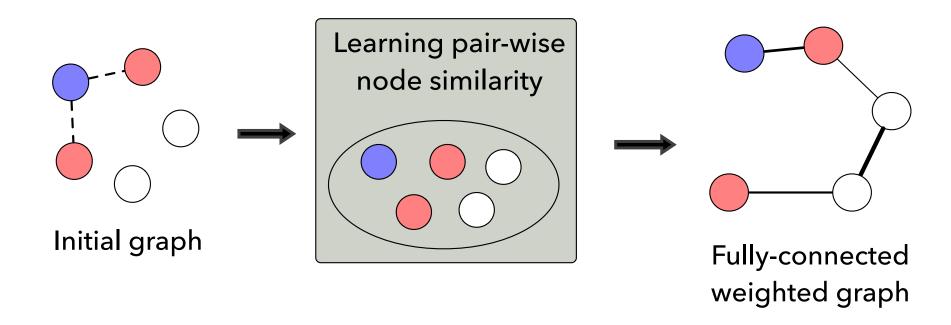
## Kernel-based Similarity Metric Functions





### Structure-aware Similarity Metric Learning

- Learning a weighted adjacency matrix by computing the pair-wise node similarity in the embedding space
- Considering existing edge information of the intrinsic graph in addition to the node information



#### Structure-aware Attention Mechanism

#### Variant 1)

$$S_{i,j}^l = \operatorname{softmax}(\mathbf{u}^T \operatorname{tanh}(\mathbf{W}[\mathbf{h}_i^l, \mathbf{h}_j^l, \mathbf{v}_i, \mathbf{v}_j, \mathbf{e}_{i,j}]))$$

Initial graph

Variant 2)

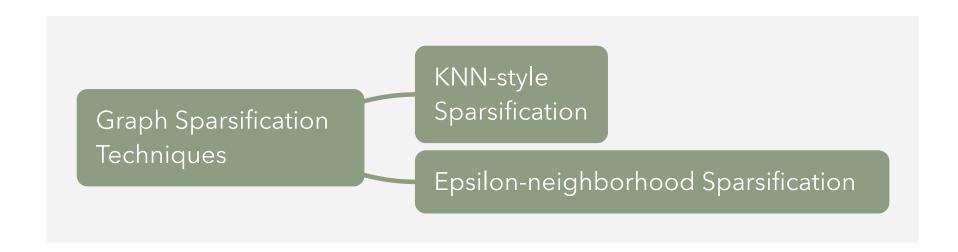
$$S_{i,j} = \frac{\text{ReLU}(\mathbf{W}^{Q}\mathbf{v}_{i})^{T}(\text{ReLU}(\mathbf{W}^{K}\mathbf{v}_{i}) + \text{ReLU}(\mathbf{W}^{R}\mathbf{e}_{i,j}))}{\sqrt{d}}$$

Edge embeddings

Fully-connected weighted graph

### **Graph Sparsification Techniques**

- Similarity metric functions learn a fully-connected graph
- Fully-connected graph is computationally expensive and might introduce noise
- Enforcing sparsity to the learned graph structure
- Various techniques



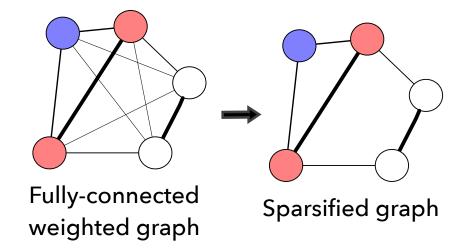
### Common Graph Sparsification Options

Option 1) KNN-style Sparsification

$$\mathbf{A}_{i,:} = \operatorname{topk}(\mathbf{S}_{i,:})$$

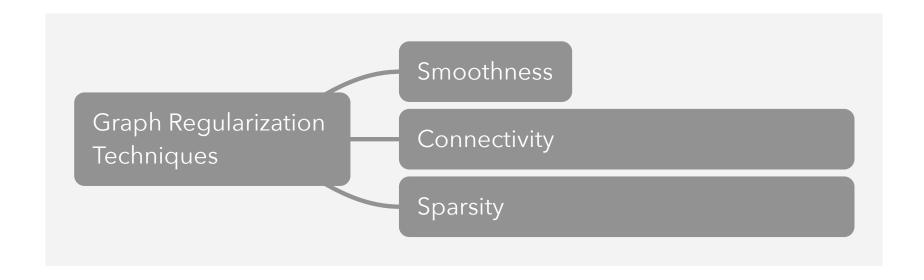
Option 2) epsilon-neighborhood Sparsification

$$A_{i,j} = \begin{cases} S_{i,j} & S_{i,j} > \varepsilon \\ 0 & \text{otherwise} \end{cases}$$



#### Graph Regularization Techniques

- Enforcing common graph properties to the learned graph structure
- Combining both task prediction loss and graph regularization loss
- Various graph properties



#### **Graph Regularization Techniques**

Smoothness

$$\Omega(\mathbf{A}, \mathbf{X}) = \frac{1}{2n^2} \sum_{i,j} A_{i,j} ||\mathbf{X}_i - \mathbf{X}_j||^2 = \frac{1}{n^2} \operatorname{tr}(\mathbf{X}^\top \mathbf{L} \mathbf{X})$$

Connectivity

$$\frac{-1}{n} \mathbf{1}^{\top} \log(\mathbf{A} \mathbf{1})$$

Sparsity

$$\frac{1}{n^2}||\mathbf{A}||_F^2$$

Borrowed from unsupervised GSL from smooth signals!

### Combining Intrinsic and Implicit Graph Structures

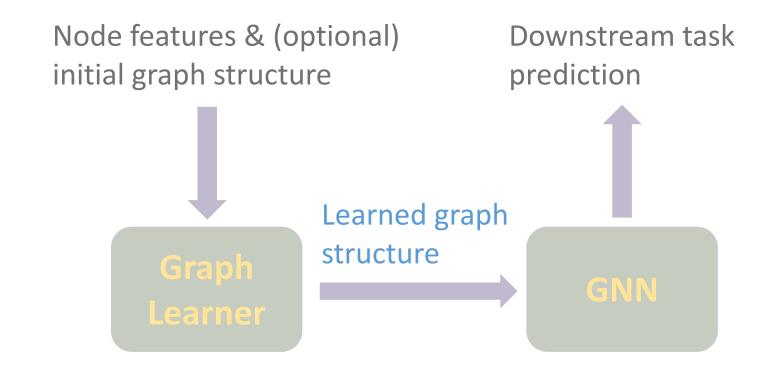
- Intrinsic graph typically still carries rich and useful information
- Learned implicit graph is potentially a "shift" (e.g., substructures) from the intrinsic graph structure

$$\widetilde{A} = \lambda L^{(0)} + (1 - \lambda)f(A)$$

Normalized graph Laplacian

f(A) can be arbitrary operation, e.g., graph Laplacian, row-normalization

### Learning Paradigms: Joint Learning



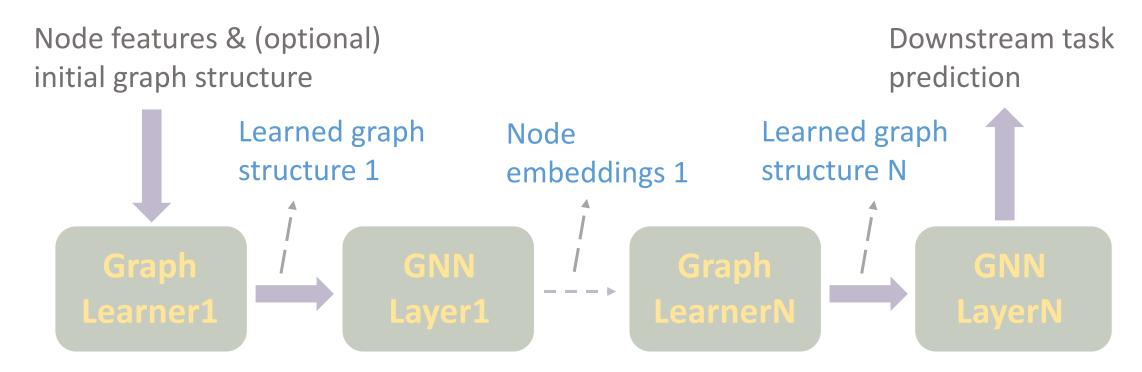
Chen at al. "GraphFlow: Exploiting Conversation Flow with Graph Neural Networks for Conversational Machine Comprehension". IJCAI 2020.

Chen et al. "Reinforcement Learning Based Graph-to-Sequence Model for Natural Question Generation". ICLR 2020.

Liu et al. "Contextualized Non-local Neural Networks for Sequence Learning". AAAI 2019.

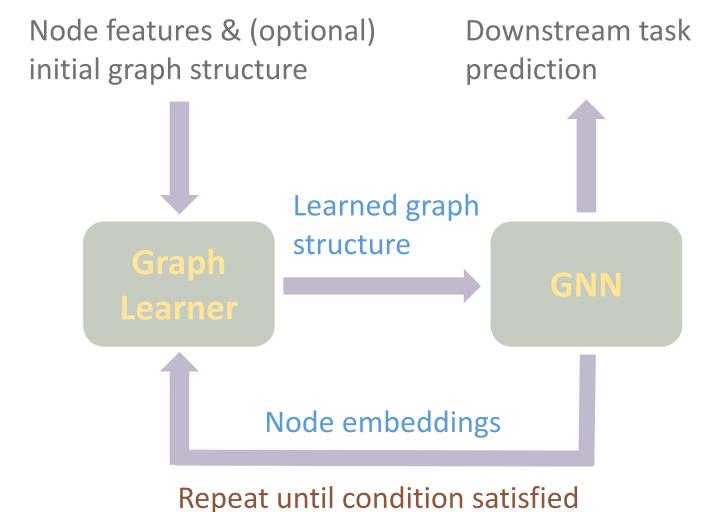
Liu et al. "Retrieval-Augmented Generation for Code Summarization via Hybrid GNN". ICLR 2021.

### Learning Paradigms: Adaptive Learning

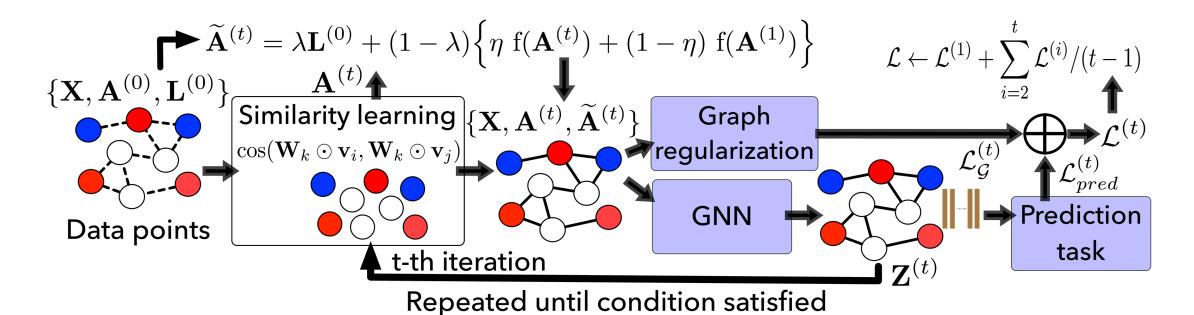


Repeat for fixed num. of stacked GNN layers

## Learning Paradigms: Iterative Learning



## Iterative Deep Graph Learning [Chen et al., NeurIPS 2020]

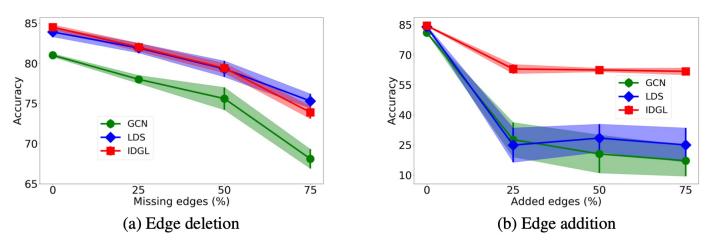


- GSL as similarity metric learning
- Graph regularization to control smoothness, sparsity and connectivity
- Iterative method to refine the graph structure and graph embeddings
- Better scalability (O(n^2) -> O(n)) using anchor-based approximation technique

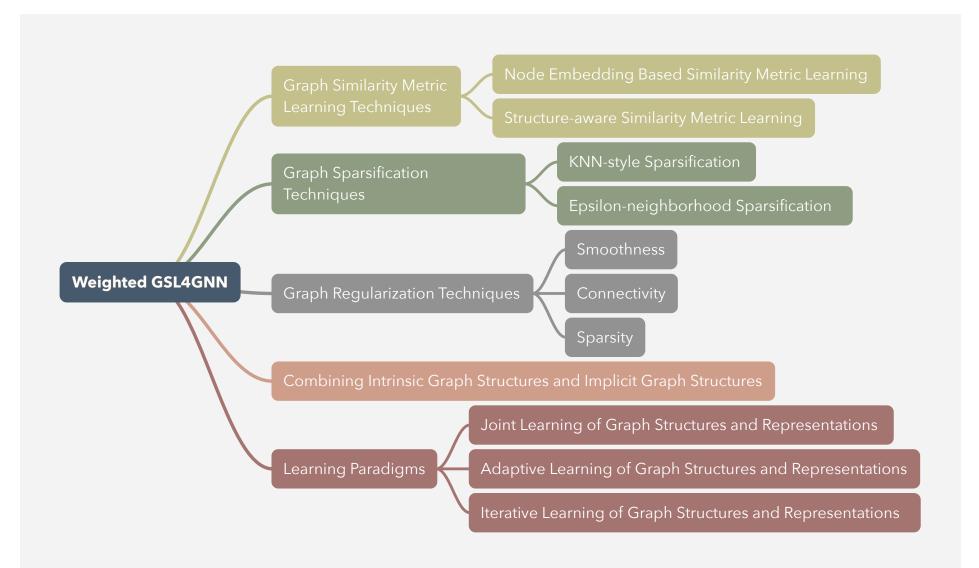
# Iterative Deep Graph Learning [Chen et al., NeurIPS 2020]

Model	Cora	Citeseer	Pubmed	ogbn-arxiv	Wine	Cancer	Digits
GCN	81.5	70.3	79.0	71.7 (0.3)			_
GAT	83.0 (0.7)	72.5 (0.7)	79.0 (0.3)				
GraphSAGE	77.4 (1.0)	67.0 (1.0)	76.6 (0.8)	71.5 (0.3)	_	_	_
APPNP	<u> </u>	<b>75.7</b> ( <b>0.3</b> )	79.7 (0.3)				
H-GCN	84.5 (0.5)	72.8 (0.5)	79.8 (0.4)		_	_	
GCN+GDC	83.6 (0.2)	73.4 (0.3)	78.7 (0.4)		_	_	_
LDS	84.1 (0.4)	75.0 (0.4)			97.3 (0.4)	94.4 (1.9)	92.5 (0.7)
GCN <sub>kNN</sub> *	_	_	_	_	95.9 (0.9)	94.7 (1.2)	89.5 (1.3)
$GAT_{kNN}*$	_	_	_		95.8 (3.1)	88.6 (2.7)	89.8 (0.6)
GraphSAGE <sub>kNN</sub> *	<u> </u>	_	_		96.5 (1.1)	92.8 (1.0)	88.4 (1.8)
LDS*	83.9 (0.6)	74.8 (0.3)			96.9 (1.4)	93.4 (2.4)	90.8 (2.5)
IDGL	84.5 (0.3)	74.1 (0.2)	_		97.8 (0.6)	95.1 (1.0)	93.1 (0.5)
IDGL-ANCH	84.4 (0.2)	72.0 (1.0)	83.0 (0.2)	72.0 (0.3)	98.1 (1.1)	94.8 (1.4)	93.2 (0.9)

#### Node classification results.



## Weighted GSL4GNN Summary



#### **Connections to Other Problems**

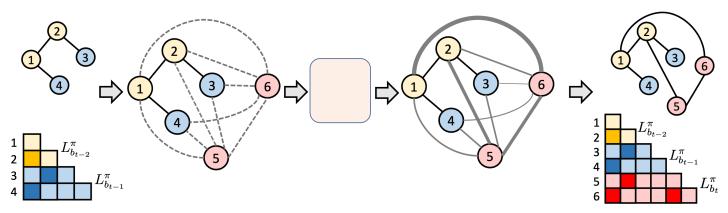
# Connections to Other Problems: GSL as Graph Generation

#### **Connections:**

Learning graphs from data

#### Differences:

- Graph generation: generating new graphs where both nodes and edges are added by sampling from the learned graph distribution.
- GSL: learning a graph structure given a set of node attributes.



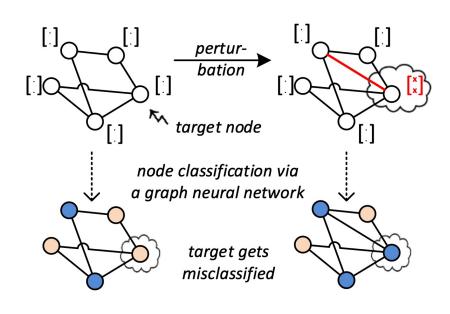
# Connections to Other Problems: GSL for Graph Adversarial Defenses

#### **Connections:**

- Improving potentially error-prone (e.g., noisy or incomplete) input graphs
- Graph adversarial defenses can benefit from GSL techniques

#### Differences:

- Graph adversarial defenses: initial graph structure is available, but potentially poisoned by adversarial attacks
- GSL: initial graph structure is available or unavailable



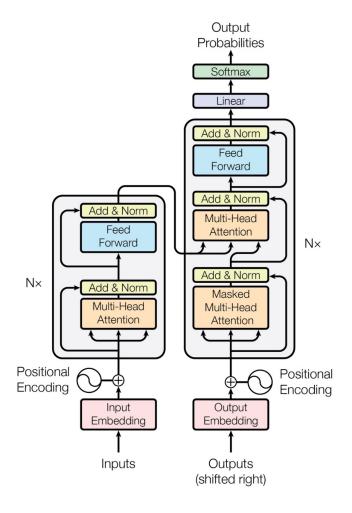
#### Connections to Other Problems: Transformers

#### **Connections:**

 Transformer models aim to learn a self-attention matrix between every pair of objects adaptively at each layer, similar to adaptive learning paradigm for weighted GSL

#### Differences:

 Vanilla Transformers don't handle graph-structured data (graph transformers combining transformers and GNNs)



# **GSL4GNN:** Future Directions and Conclusions

#### **Future Directions**

- Robust GSL
  - Noisy initial graph structures and noisy node attributes
- Scalable GSL
  - Pair-wise node similarity computation is expensive and intractable for large graphs
  - Potential solutions: LSH/low-rank/random feature/kernel methods
- GSL for Heterogeneous Graphs
  - Heterogeneous graphs carry on richer information
  - Less explored

#### Conclusions

- GNNs are powerful machine learning tools for modeling graphstructured data
- GSL has been extensively studied in traditional machine learning
- GSL4GNN is a trending research area and critical for the success of GNN applications
- Open challenges in GSL4GNN

#### Resources

- Chen, Yu, and Lingfei Wu. "Graph Neural Networks: Graph Structure Learning." Graph Neural Networks: Foundations, Frontiers, and Applications. Springer, Singapore, 2022. 297-321. (website, video)
- Zhu, Yanqiao, et al. "Deep graph structure learning for robust representations: A survey." arXiv preprint arXiv:2103.03036 (2021).
- Dong, Guimin, et al. "Graph Neural Networks in IoT: A Survey." arXiv 2022.
- Wu, Lingfei, et al. "Deep Learning on Graphs for Natural Language Processing." Tutorials at NAACL'21, SIGIR'21, KDD'21, IJCAI'21, AAAI'22 and TheWebConf'22. (website)

# Thanks! Q&A